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Reprinted from February 1974 issue of

SOCIETY OF PETROLEUM ENGINEERS JOURNAL

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ABSTRACT

Specimens of Salem limestone were loaded cyclicly at a frequency of 2 cycles/sec in uniaxial cyclic compression, tension, and compression-tension. The number of cycles to failure, maximum deformation for each cycle, and load-deformation hysteresis loops were recorded. The fatigue life and fatigue limit values under cyclic compressive loading are comparable with those under cyclic tensile loading, whereas under cyclic compressive-tensile loading they are considerably lower.

INTRODUCTION

The study of rock behavior in cyclic loading has been relatively ignored in the past, even though certain problems in rock mechanics are closely related to cyclic loading.[1,3] These problems include the effects of percussive drilling and the vibrations generated by blasting. An understanding of the mechanisms of fatigue failure in rock can be expected to help improve drilling efficiency and prevent vibration damage caused by blasting.

Because of the lack of basic information on rock behavior under cyclic loading, the Federal Bureau of Mines, Twin Cities Mining Research Center began in 1968 an extensive program for studying cyclic loading effects. This program included the investigation of the behavior of rock loaded cyclicly at different frequencies under varying test geometries, loading configurations, and environments. In the high-frequency range, sonic power transducers are being used to apply cyclic loading at a frequency of 10,000 Hz, and an electromagnetic shaker is being used at frequencies from 100 to 1,000 Hz. In the low-frequency range, cyclic loading of 2 to 10 Hz is applied by a closed-loop servocontrolled electrohydraulic testing machine. In each frequency range, experiments are conducted to provide the following information: fatigue limits, fatigue life, energy dissipation, temperature induced in the specimen, and the time history of load and deformation.

This paper presents the first phase of the results obtained on specimens of Salem limestone loaded in the low-frequency range. The early findings on the high-frequency effects were reported separately.[2] Recently, the effect of cyclic loading on rock behavior has been receiving more attention and considerable information is being generated.[4-7]

GENERAL LOADING CONCEPT IN CYCLIC LOADING

In conventional strength tests the monotonic loading program is specified by the loading rate and control mode. For cyclic loading, where the load is a periodic function of time, the problem is more complex. To evaluate such material properties as fatigue life, the load must be described systematically and concisely in terms of physically significant parameters.

For a general case, one approach is to divide the cyclic stress into time-independent and time-dependent components. The time-independent component $\bar{\sigma}$ (or mean stress) is the time average of the stress. A cyclic stress with an amplitude σ_A and zero mean can be superimposed on this loading. For the usual case of cyclic loading with steady loading conditions, the stress can be described as follows.

$$\sigma = \bar{\sigma} + \sigma_A f(t) , \quad \ldots \ldots \quad (1)$$

where $f(t)$ is a periodic function of time, t, and can be represented by a sine or sawtooth wave.

Other ways of describing the stress are available such as using the maximum and minimum stresses, which are related to the mean and amplitude:

$$\sigma_{max} = \bar{\sigma} + \sigma_A \quad \ldots \ldots \quad (2)$$

and

$$\sigma_{min} = \bar{\sigma} - \sigma_A \quad \ldots \ldots \quad (3)$$

The key issue is to describe the loading in terms that will correlate with the material properties of interest. The use of amplitude and mean stress to describe cyclic loading separates the time-dependent from the time-independent portion of the stress because the effect of each portion of the loading should be investigated separately.

In analyzing the effect of cyclic loading on rock, another significant factor is the large difference between the tensile strength and the compressive strength. Although stresses in the tensile region are small, they are significant in terms of fatigue

Paper (SPE 4249) was presented at SPE-AIME 6th Conference on Drilling and Rock Mechanics, held in Austin, Tex., Jan. 22-23, 1973.

[1]References listed at end of paper.

damage. One way to bring into balance the effects of tensile and compressive stresses is to use a normalized stress. This can be done by dividing the compressive stress by the compressive strength, C, and the tensile stress by the tensile strength, T. Thus, the compressive stress and the tensile stress will be represented in common terms.

This experimental study investigated the effect of cyclic compressive loading, cyclic tensile loading, and cyclic compressive-tensile loading. The tensile and compressive stresses were applied as percentages of the strengths. In the cyclic compressive-tensile loading tests, the sample was cycled between a given percentage of the compressive strength and the same percentage of the tensile strength. In this particular case, data can be examined in terms of the stress amplitude or mean stress since they are interrelated by the strengths:

$$\sigma_A = \frac{C - T}{C + T} \bar{\sigma} . \quad (4)$$

EXPERIMENTAL PROGRAM

The Salem limestone chosen for this study consists of a high-porosity aggregate of fossil and öolite particles.[8] Eighty-five 2-in. cylindrical specimens were tested. Ten specimens — five each — were used to obtain the uniaxial compressive and tensile strength. The compressive and tensile strengths were determined by monotonic loading at a constant deformation rate of 100 mm/sec corresponding to a cyclic loading at a frequency of 2 Hz (2 cycles/sec) with a stress amplitude of 10,000 psi. The strengths obtained are 10,200 psi for compressive strength, C, and 600 psi for tensile strength, T.

A closed-loop servocontrolled electrohydraulic machine was used to test monotonic and cyclic loading. Its operational principles have already been described.[9] The test setup for cyclic compressive loading tests used steel spacers 2 in. in diameter and 2 in. long inserted between specimen ends and machine platens for proper end conditions. This setup is like that used in testing monotonic compressive rock strength.

To adapt the machine to applying cyclic tensile and compressive-tensile loading, a special fixture was designed. The test fixture made of steel was 2 in. in diameter and 7 in. long, with the middle 3-in. section reduced to 1.4 in. in diameter. The end of the test fixture was 2 in. in diameter by 2 in. long. This design made the fixture compatible with the specimen and insured a more uniform stress distribution in the specimen.[10] In setting up the tests, the fixtures were first inserted into the top and bottom machine platens with a space between the fixtures approximately equal to the length of the specimen. After both ends of the specimen were given a thin coat of rapid-bonding adhesive, the specimen was inserted between the top and bottom test fixtures; the top and bottom fixtures were then brought into direct contact with the specimen. The adhesive dried and cured very quickly with a tensile strength of approximately 3,000 psi.

To test the accuracy of alignment, four strain gauges 90° apart were mounted axially in the middle of the specimen to detect any bending moment. The maximum deviation of strain readings for the four strain gauges was about 6 percent from the average strain for loading levels up to 60 percent of the ultimate strength. Similar measurements were performed on an epoxy specimen, and the maximum deviation in strain reading was 0.15 percent. Since the larger strain deviation in the rock specimen can be attributed to local inhomogeneity of microstructure, the specimen-mounting technique was considered appropriate.

In making the fatigue study, tests were conducted in a load-control mode with a constant cyclic rate of 2 Hz. Clip-on gauges 1 in. long were clamped in the middle of the specimen to measure deformation. The load-deformation, load-time, and deformation-time curves were recorded simultaneously throughout the tests. A thermocouple was also mounted on the surface of selected specimens to measure heat generated in the specimen during cyclic testing. No noticeable temperature change was recorded in any test. Specimens were loaded cyclicly up to 10^6 cycles. If the specimen did not break within this range, the test was terminated. To keep the specimen in direct contact with the steel spacers under cyclic compressive loading, the lower limit of the stress amplitude was maintained at 50 psi instead of zero. Under cyclic tension, however, the lower stress limit was maintained at zero because the specimen was cemented to the test fixtures.

In the cyclic compressive loading tests, the first loading cycle consisted of loading from unloaded condition to the specified maximum compressive stress, σ_{max}, and then unloading to 50 psi. From the second cycle on, the applied stress cycled between 50 psi and the maximum stress. Twenty-five specimens were tested at maximum stresses ranging from 70 to 95 percent of the compressive strength.

The cyclic tensile loading was performed between the unloaded condition and maximum tensile stress, σ_{min}. Twenty-one specimens were used and subjected to maximum tensile stresses ranging from 50 to 95 percent of the tensile strength.

In the cyclic compressive-tensile loading, the

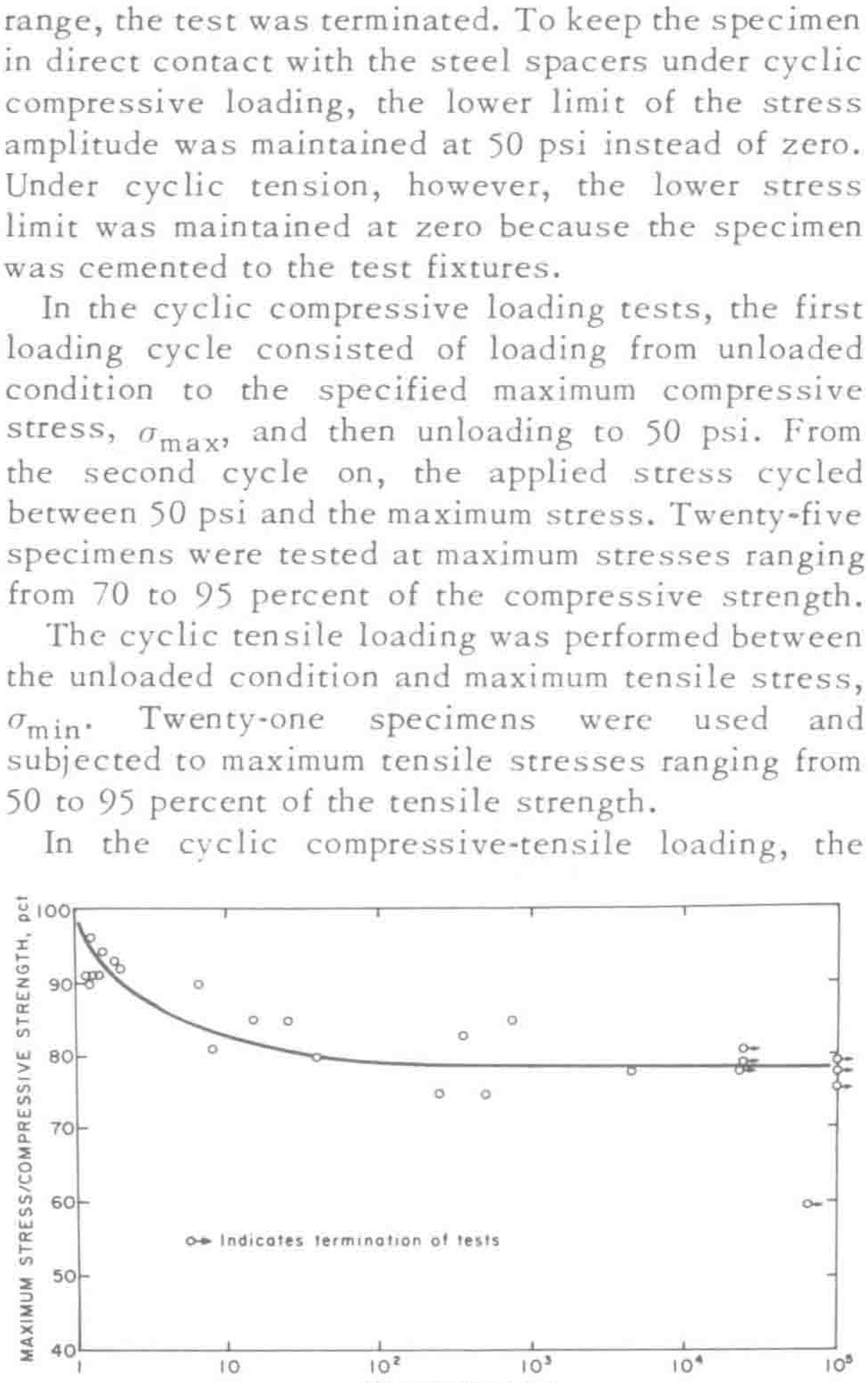


FIG. 1 — FATIGUE CURVE FOR SALEM LIMESTONE UNDER CYCLIC COMPRESSIVE LOADING.

stress amplitudes and mean stress were so chosen that the percentage of maximum compressive stress to compressive strength was the same as that of maximum tensile stress to tensile strength. Since the ratio of compressive strength to tensile strength for Salem limestone is about 17, such an arrangement of maximum compression and tension always results in a compressive mean stress. Therefore, the tests were conducted at mean stresses ranging from 25 to 45 percent of the compressive strength, and all tests were begun with the compressive portion of the cycle.

EXPERIMENTAL RESULTS

CYCLIC COMPRESSIVE LOADING

The fatigue curve in cyclic compression is shown in Fig. 1 as the ratio of maximum stress, σ_{max}, to compressive strength vs the number of loading cycles. Although the data are scattered, it can be approximated by an exponentially decaying curve with a fatigue limit at about 78 percent of the compressive strength. A typical record of hysteresis loops is shown in Fig. 2; the cycle number and the area for each loop are indicated. The origin of each loop is switched to a new position so that the loops do not overlap and smear the loop shape and size. The area of the hysteresis loop sharply decreases during the first few cycles and rapidly reaches a constant level until a few cycles before complete failure, at which time it increases sharply. Initially, the maximum deformation of the specimen for each cycle is nearly constant, but it increases exponentially near the fatigue-failure point.

CYCLIC TENSILE LOADING

The fatigue curve (Fig. 3) for cyclic tension is a slowly decaying curve with the fatigue limit at about 70 percent of the tensile strength.

A typical record of the shape and size of hysteresis loops for several selected cycles is shown in Fig. 4. The area of the hysteresis loop, as in cyclic compressive loading, decreases rapidly at the beginning of cyclic loading and levels off later on. There is no sign of gradual increase in the size of hysteresis loop as the specimen approaches failure. The maximum deformation of the specimen for each cycle is constant after an initial decrease. The fracture of the specimen appears to be instantaneous.

CYCLIC COMPRESSIVE-TENSILE LOADING

The endurance under cyclic compressive-tensile loading was much less than that under either cyclic compressive loading or cyclic tensile loading. Specimens failed between 5 and 106 cycles and predominantly at the peak of the compressive cycle. Although the data points are as scattered as those from the cyclic tensile loading tests, the fatigue curve can again be approximated by a downward curve (Fig. 5). A typical record of the shape and size of the hysteresis loops is shown in Fig. 6. The area of the loop decreases rapidly in the first few cycles and then increases exponentially near failure. The maximum deformation for each tensile half-cycle remains constant throughout the test, but that for each compressive half-cycle remains constant initially and increases exponentially as the

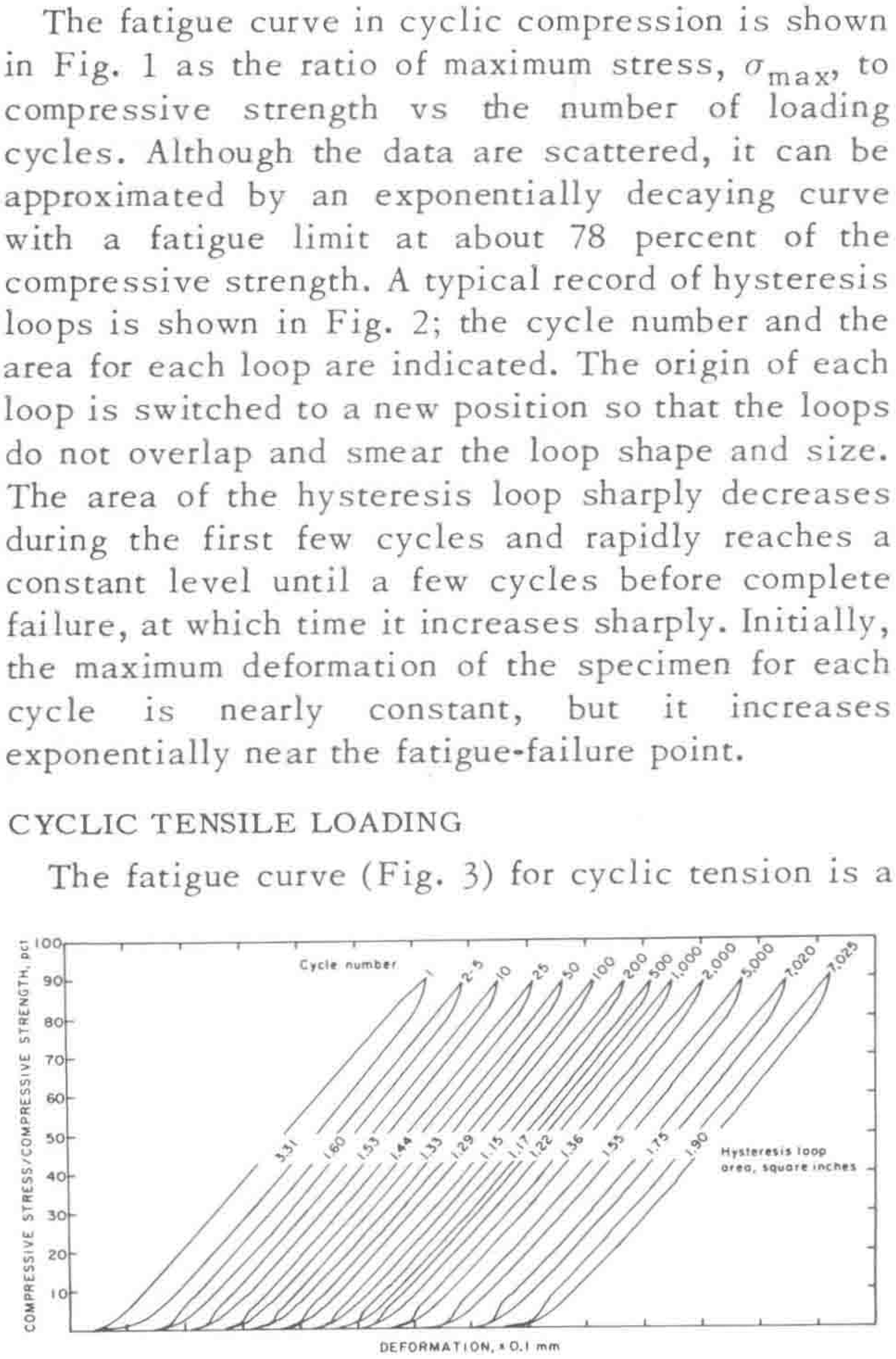


FIG. 2 — HYSTERESIS LOOPS OF SELECTED CYCLES FOR SALEM LIMESTONE UNDER CYCLIC COMPRESSIVE LOADING.

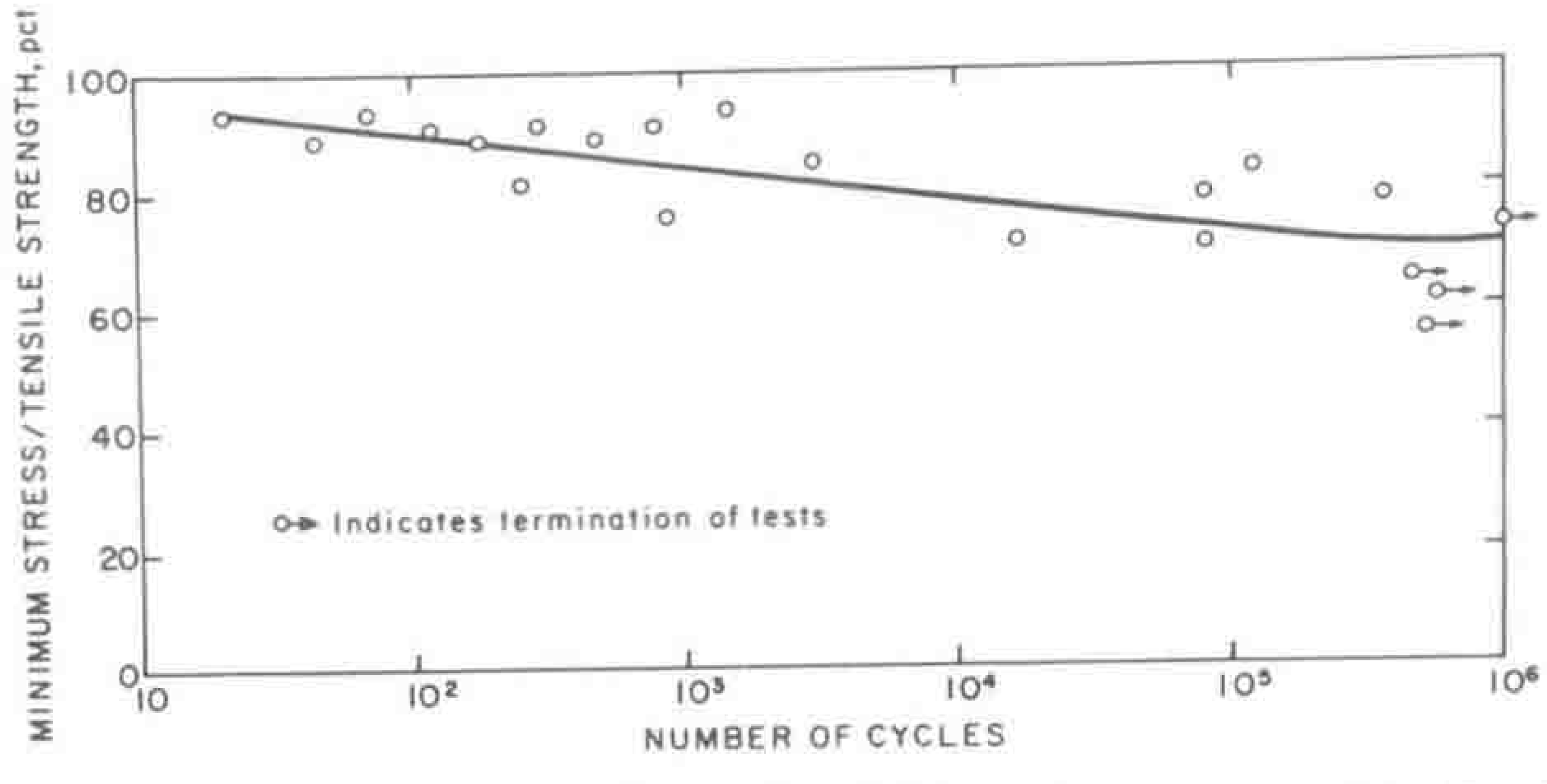


FIG. 3 — FATIGUE CURVE FOR SALEM LIMESTONE UNDER CYCLIC TENSILE LOADING.

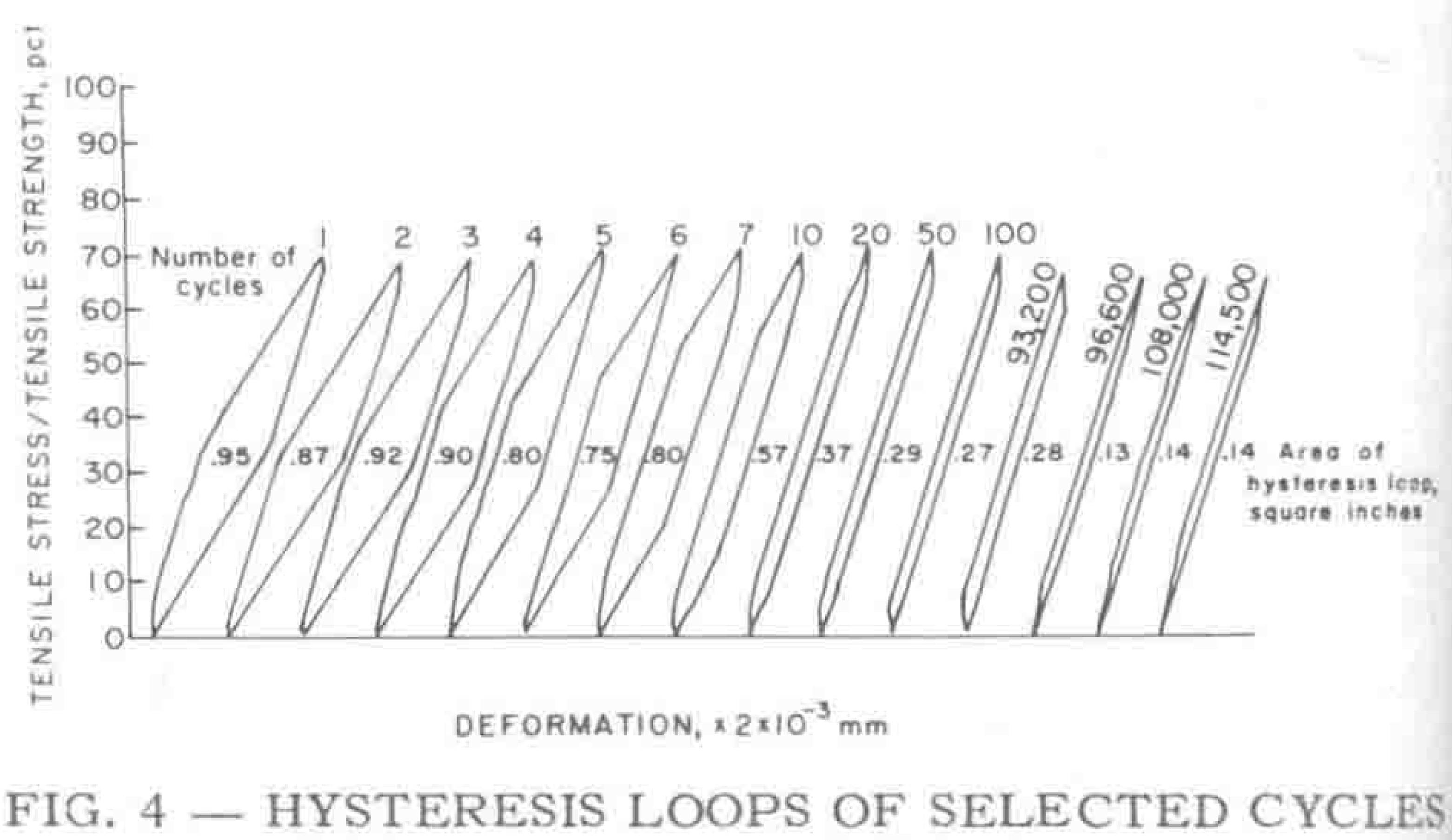


FIG. 4 — HYSTERESIS LOOPS OF SELECTED CYCLES FOR SALEM LIMESTONE UNDER CYCLIC TENSILE LOADING.

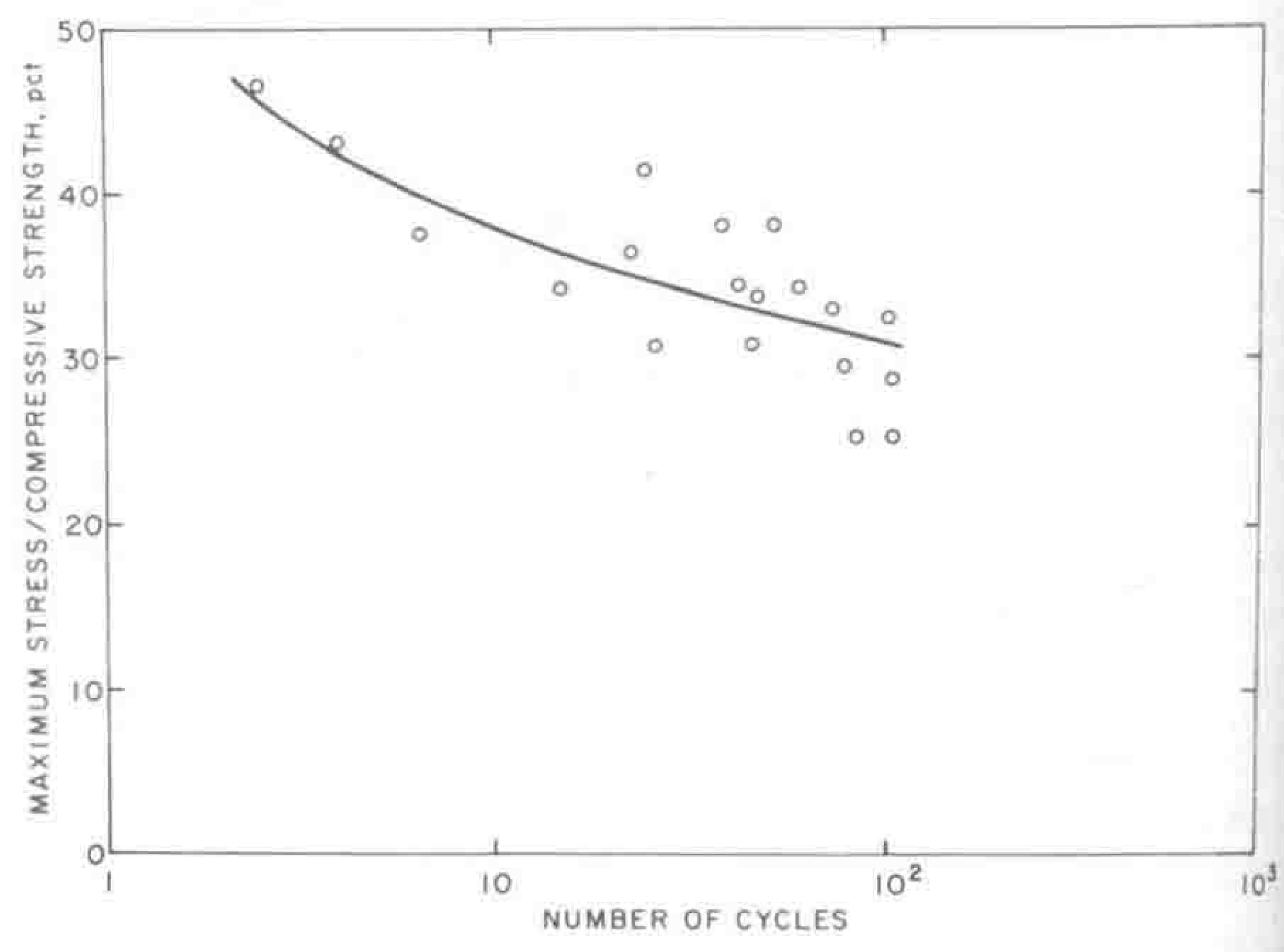


FIG. 5 — FATIGUE CURVE FOR SALEM LIMESTONE UNDER CYCLIC COMPRESSIVE-TENSILE LOADING.

specimen approaches failure. This behavior is similar to the combination of the individual responses to cyclic compressive and cyclic tensile loading.

DISCUSSION

FATIGUE LIFE

The results for Salem limestone shown in Figs. 1 and 3 indicate that the fatigue limits under both cyclic compressive or tensile loading are at 78 and 70 percent of compressive and tensile strength, respectively, and the fatigue lives under both are comparable when loaded to the same percentage of strength. The fatigue life under cyclic compressive-tensile loading, however, is very much shorter. Here the data are not sufficient to establish a fatigue limit.

The fatigue life of a rock specimen can be investigated from phenomenological point of view by considering fatigue failure as owing to accumulated damage. If the deformation in the first cycle is taken as a standard deformation for that stress amplitude, the increase in deformation for each subsequent cycle can be used as a measure of degree of damage.

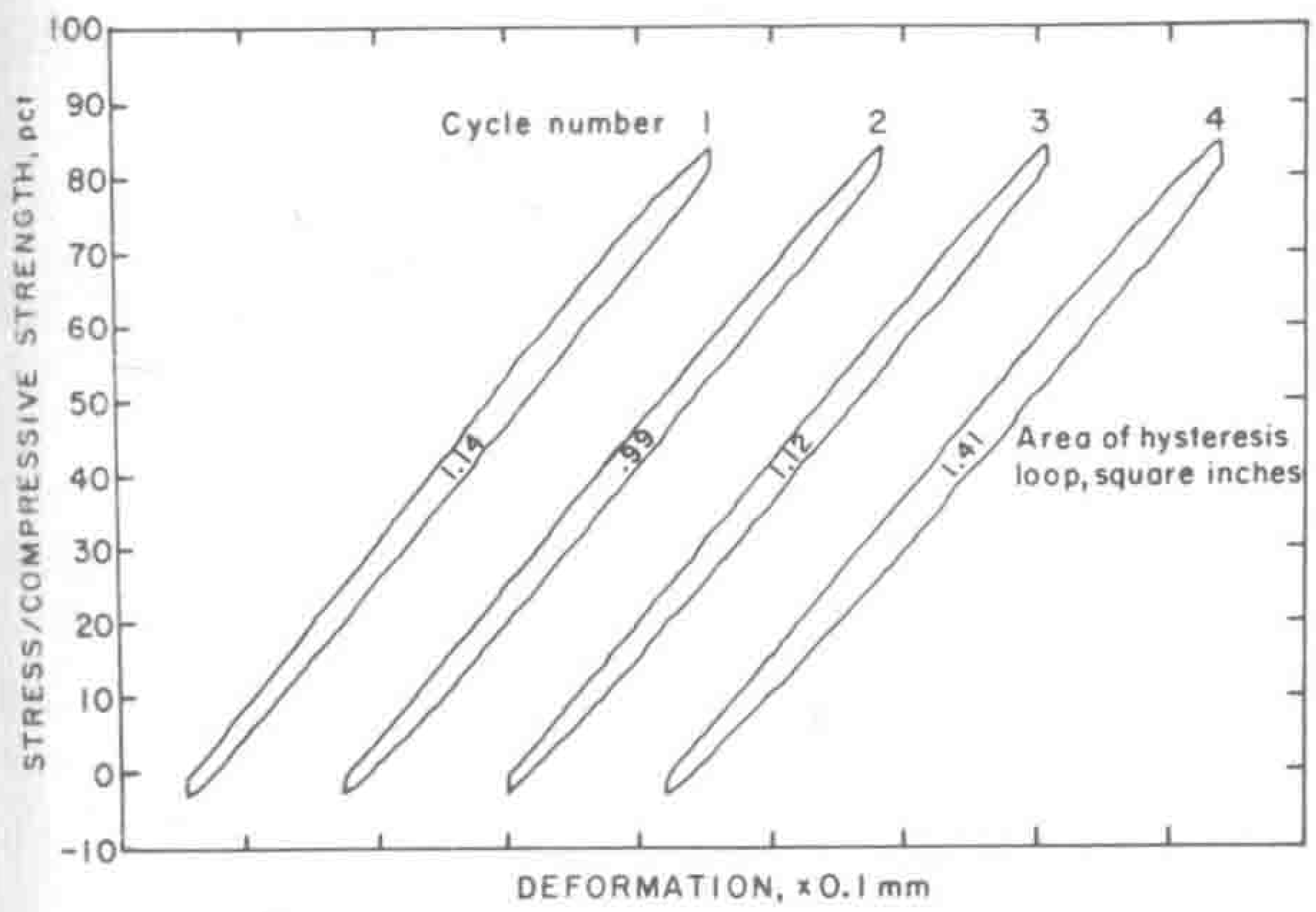


FIG. 6 — HYSTERESIS LOOPS OF SELECTED CYCLES FOR SALEM LIMESTONE UNDER COMPRESSIVE-TENSILE LOADING.

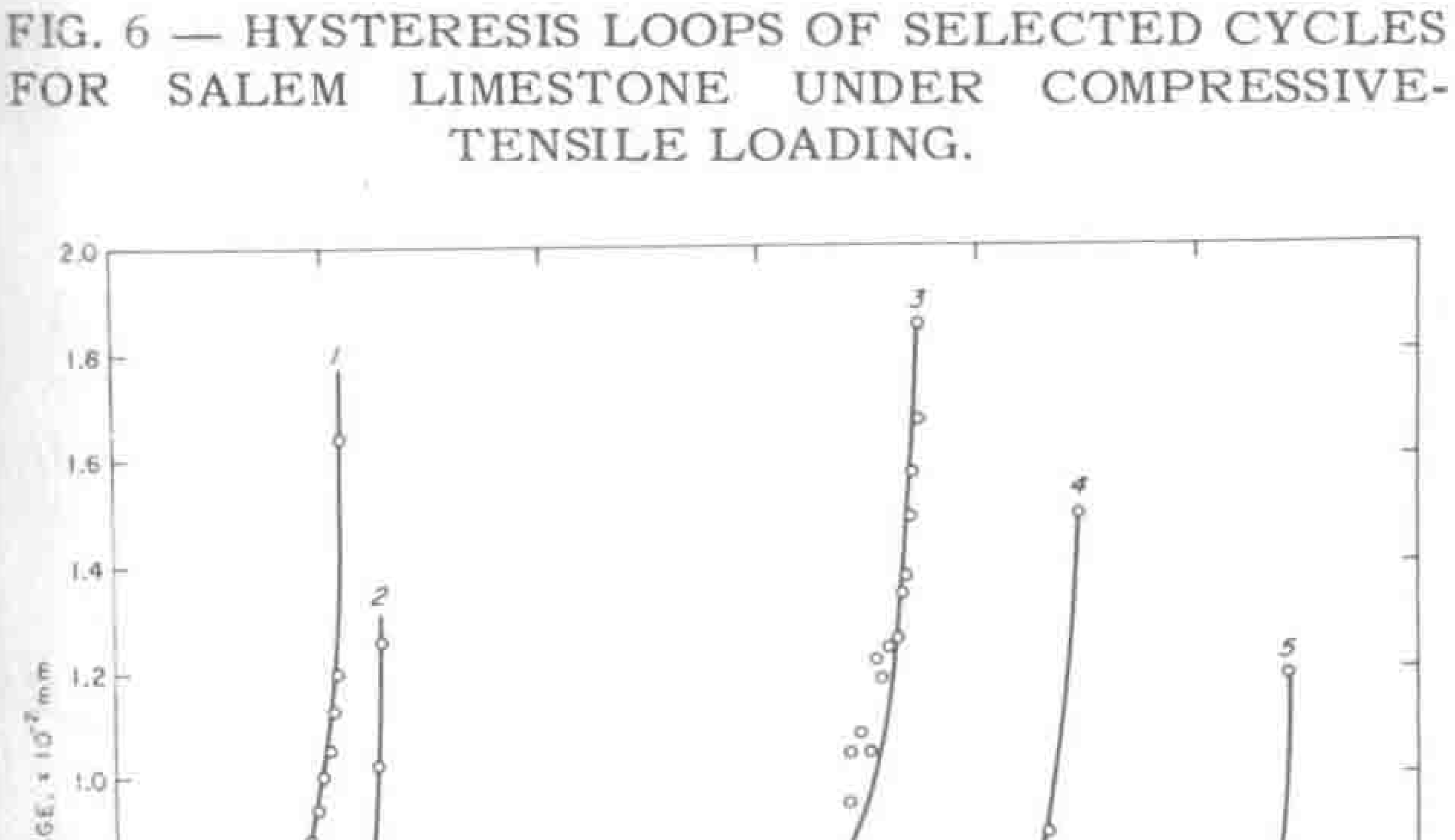


FIG. 7 — FATIGUE DAMAGE FOR CYCLIC COMPRESSIVE LOADING. THE SPECIMEN NUMBERS ARE INDICATED IN THE FIGURE WITH STRESS AMPLITUDE: NO. 1 = 9,700 PSI; NO. 2 = 9,200 PSI; NO. 3 = 9,300 PSI; NO. 4 = 8,500 PSI; NO. 5 = 8,600 PSI.

Fig. 7 shows the increase of deformation under cyclic compressive loading at different stress amplitudes. These damage curves generally start with a nearly constant portion followed by a linear increase, and then an exponential increase as the specimen approaches failure. Depending on stress amplitude, the increase in deformation starts from various loading cycles ranging from the second to the fiftieth cycle. Similar curves are obtained for the compressive part of the deformation under cyclic compressive-tensile loading (Fig. 8). The rate of increase in deformation under this loading condition is, however, much higher than that under cyclic compressive loading. In both cases, the specimens show a compressive mode of fracture (Figs. 9a and 9c) characterized by a gradual failing of the specimens. This mode of fracture is similar to that under monotonically increasing compressive loading.

On the contrary, the deformation for each cycle under cyclic tensile loading is constant near failure, and specimens appear to fail suddenly. The fracture plane is generally located in the midsection of the specimen and usually perpendicular to the loading direction (Fig. 9b). This mode of fracture is similar to that under monotonically increasing tensile loading.

A possible explanation of the mechanism of rapid fatigue failure with stress reversal in rock is offered by the two-dimensional crack model developed by Peng and Ortiz.[11] This model predicts that under compressive loading, en-echelon cracks parallel to the direction of maximum applied loading develop in a staggered manner. If a tensile loading is subsequently applied to this same specimen, tensile cracks perpendicular or subperpendicular to the tensile loading would most likely start from the boundary of these axial cracks, extend toward the intact parts between the parallel cracks, and eventually join the en-echelon cracks. Thus the en-echelon cracks coalesce much earlier under cyclic compressive-tensile loading than under cyclic compressive loading.

ENERGY DISSIPATION

A significant portion of input energy is dissipated

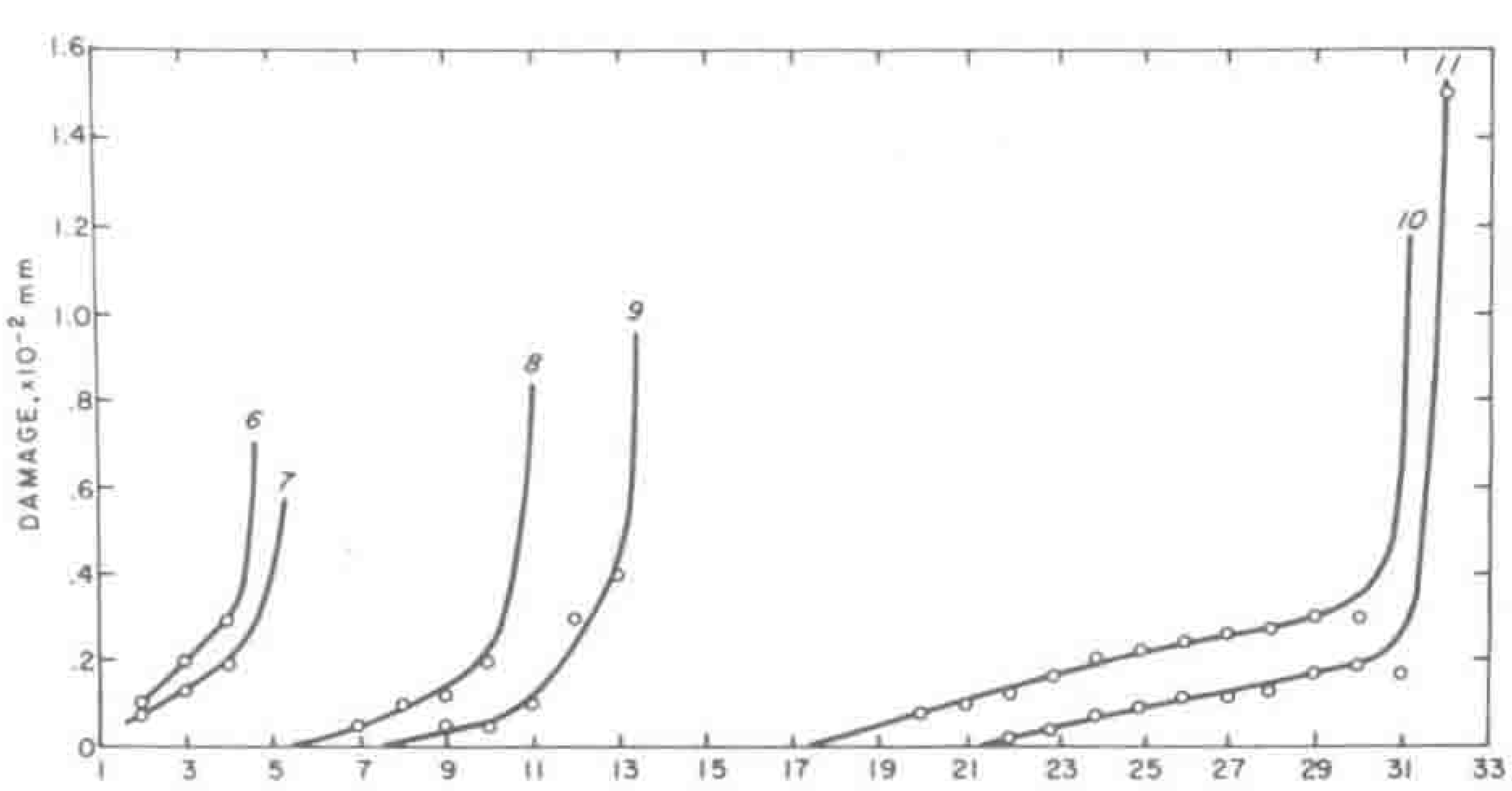


FIG. 8 — FATIGUE DAMAGE FOR CYCLIC COMPRESSIVE-TENSILE LOADING. THE SPECIMEN NUMBERS ARE INDICATED IN THE FIGURE WITH MEAN STRESS. NO. 6 = 4,100 PSI; NO. 7 = 3,400 PSI; NO. 8 = 3,800 PSI; NO. 9 = 3,000 PSI; NO. 10 = 3,800 PSI; NO. 11 = 3,400 PSI.

by the cyclic loading of rock even though it is nominally within the elastic range. The energy dissipation is determined by the area of the load-deformation curve hysteresis loops. Each type of cyclic loading produces a characteristic hysteresis loop, and these loops generally change shape and size as the load cycles accumulate.

Hysteresis loops formed under cyclic compressive loading (Fig. 2) have some of the characteristics of compressive stress-strain curves. As the load is applied, the slope increases to a steady value, which is maintained until the load is reversed. On load reversal, the load drops quickly at first and then decreases linearly until just before complete unloading, when the stiffness drops back to the initial condition. The area of the hysteresis loops that reflects energy dissipation decreases in the first few cycles and then increases again as complete failure is approached.

The amount of input energy dissipated in each load cycle can be important; for example, in predicting stress wave attenuation of energy consumption in percussive drilling. The area of the hysteresis loop, A_h, represents the energy absorbed per cycle of loading by the entire specimen — total damping energy, D_t, owing to internal or material damping.[12] The specific damping energy of the rock, D_s, represents the energy absorbed per unit volume per cycle of loading and is related to the total damping energy by the volume of the test specimen, V_c, and the stress distribution in the specimen. In this case, under cyclic loading the relationship simplifies to

$$D_s = D_t / V_c = A_h / V_c . \qquad (5)$$

This energy loss can also be expressed in terms of the damping capacity, Ψ, which is the ratio of the energy loss in the specimen to the maximum strain energy for each cycle:

$$\Psi = D_t / E_\epsilon \approx A_h / \tfrac{1}{2} P_{max} S_{max} , \qquad (6)$$

where E_ϵ = total strain energy for entire specimen at maximum displacement

$= \frac{1}{2} P_{max} S_{max}$

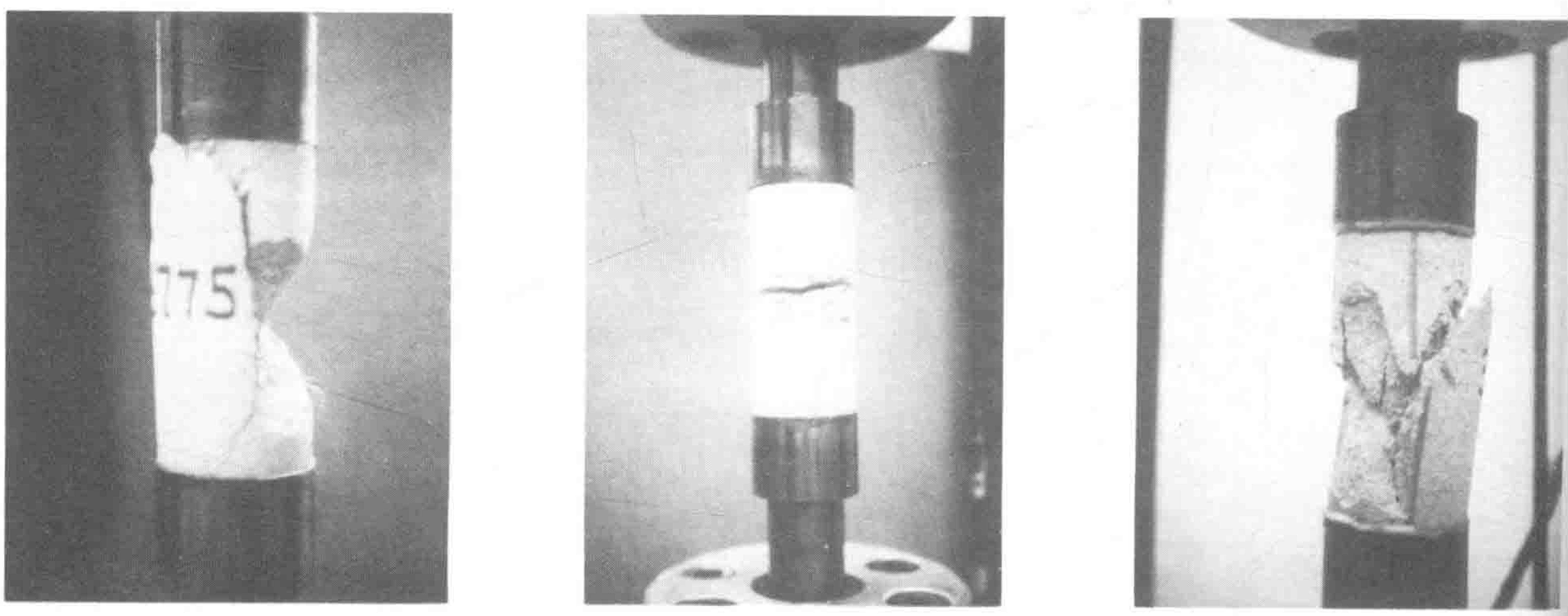
P_{max} = maximum load

S_{max} = maximum deformation

The uniaxial cyclic loading used in this program simplifies the expression for damping capacity. The total damping and strain energy terms can be replaced by the specific energy terms because of the uniform stress distribution in the specimen. Therefore the damping can be expressed in terms of specific damping capacity

$$\Psi_s = D_s / E_\epsilon = A_h / \tfrac{1}{2} P_{max} S_{max} . \qquad (7)$$

It should be pointed out that in these derivations the anisotropy and inhomogeneity of the rock and the nonuniformity in stress distribution due to the boundary conditions between the platens and the rock specimen have not been considered as significantly affecting rock damping. The choice of specific damping capacity over the specific energy was made to demonstrate more clearly the degree of energy loss during some rock fragmentation processes. That the specific damping capacity is dependent on the number of cycles can be seen in Figs. 10 through 12. The general trend is for the specific damping capacity to decrease during the first few cycles and gradually to level off later. Under cyclic compressive and compressive-tensile loading, damping capacity increases again as the specimen approaches failure, but it does not increase under cyclic tensile loading. The decrease in specific damping capacity in the first few cycles for all loading conditions may be associated with the stabilization of the internal structure of the rock due to cyclic loading. This assumption is substantiated by the change in the slope of the hysteresis loops and the increase of secant modulus as load cycles accumulate. When a specimen



a- Compression *b*- Tension *c*- Compression-Tension

FIG. 9 — MODES OF SPECIMEN FRACTURE FOR SALEM LIMESTONE UNDER DIFFERENT CYCLIC LOADING PROGRAM.

approaches failure under cyclic compressive loading, cracks develop, and the friction between the crack surfaces accounts for the increase of specific damping capacity. This behavior does not occur under cyclic tensile loading; under cyclic tensile loading, specific damping capacity does not increase as a result of sudden failure of the specimen.

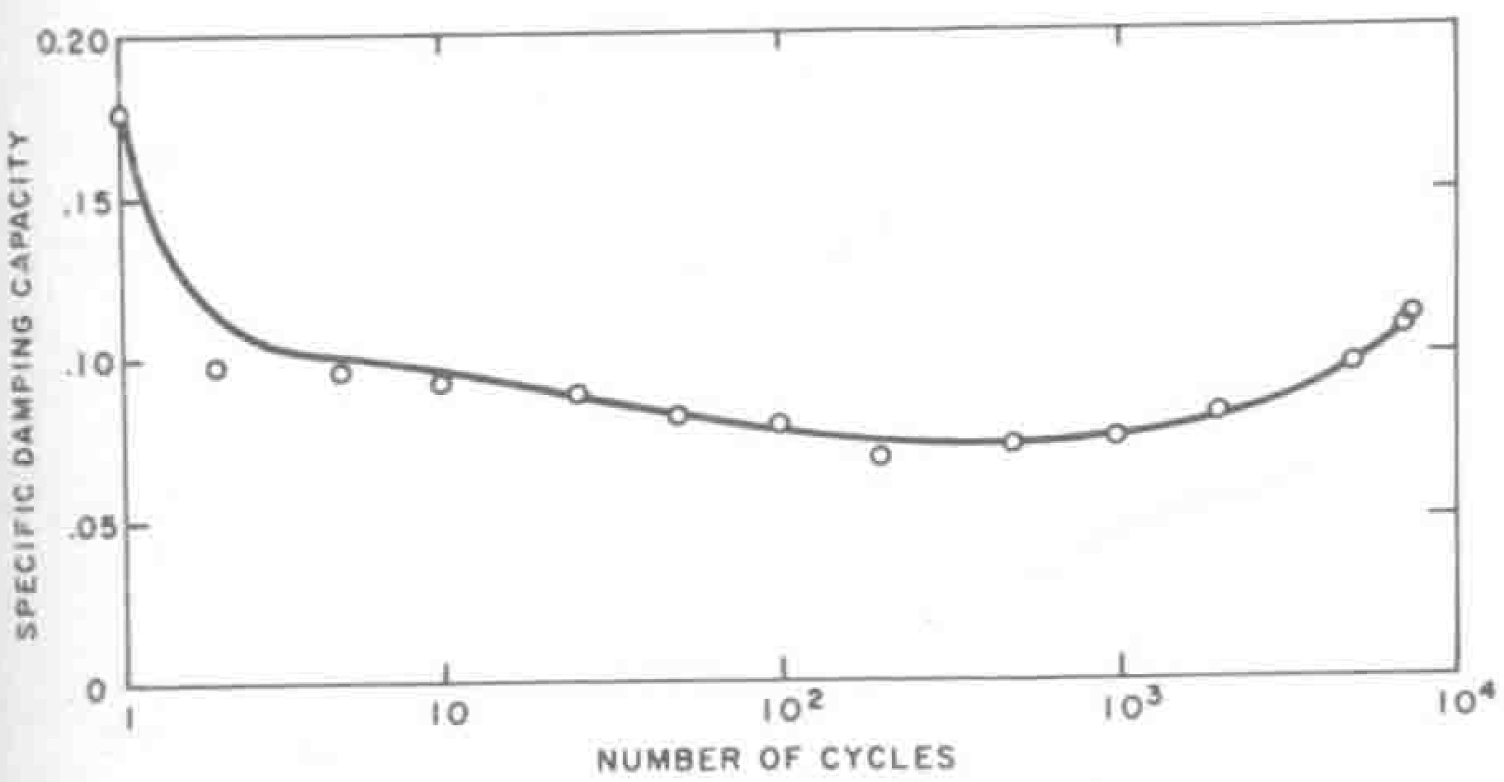


FIG. 10 — SPECIFIC DAMPING OF SALEM LIMESTONE UNDER CYCLIC COMPRESSIVE LOADING.

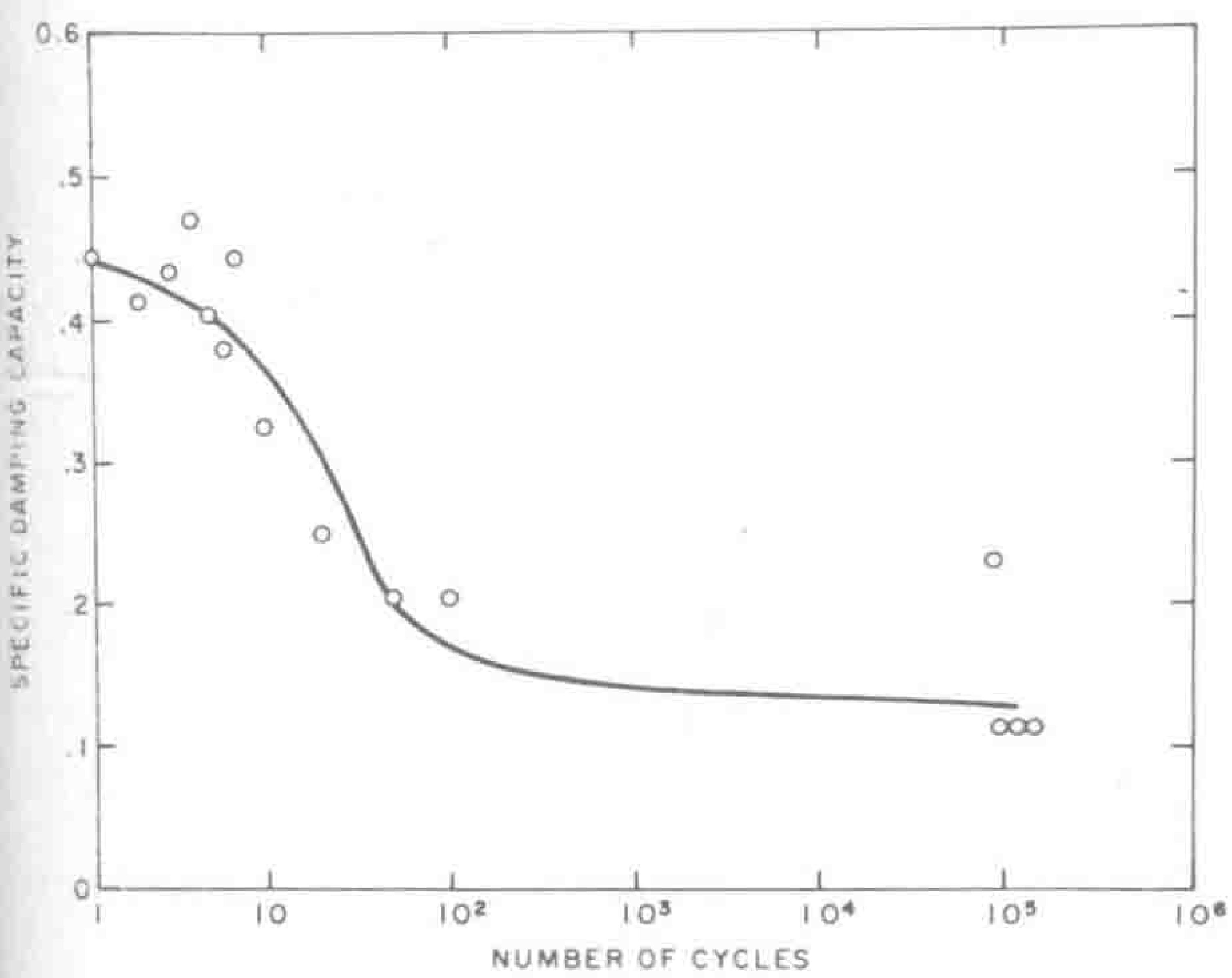


FIG. 11 — SPECIFIC DAMPING OF SALEM LIMESTONE UNDER CYCLIC TENSILE LOADING.

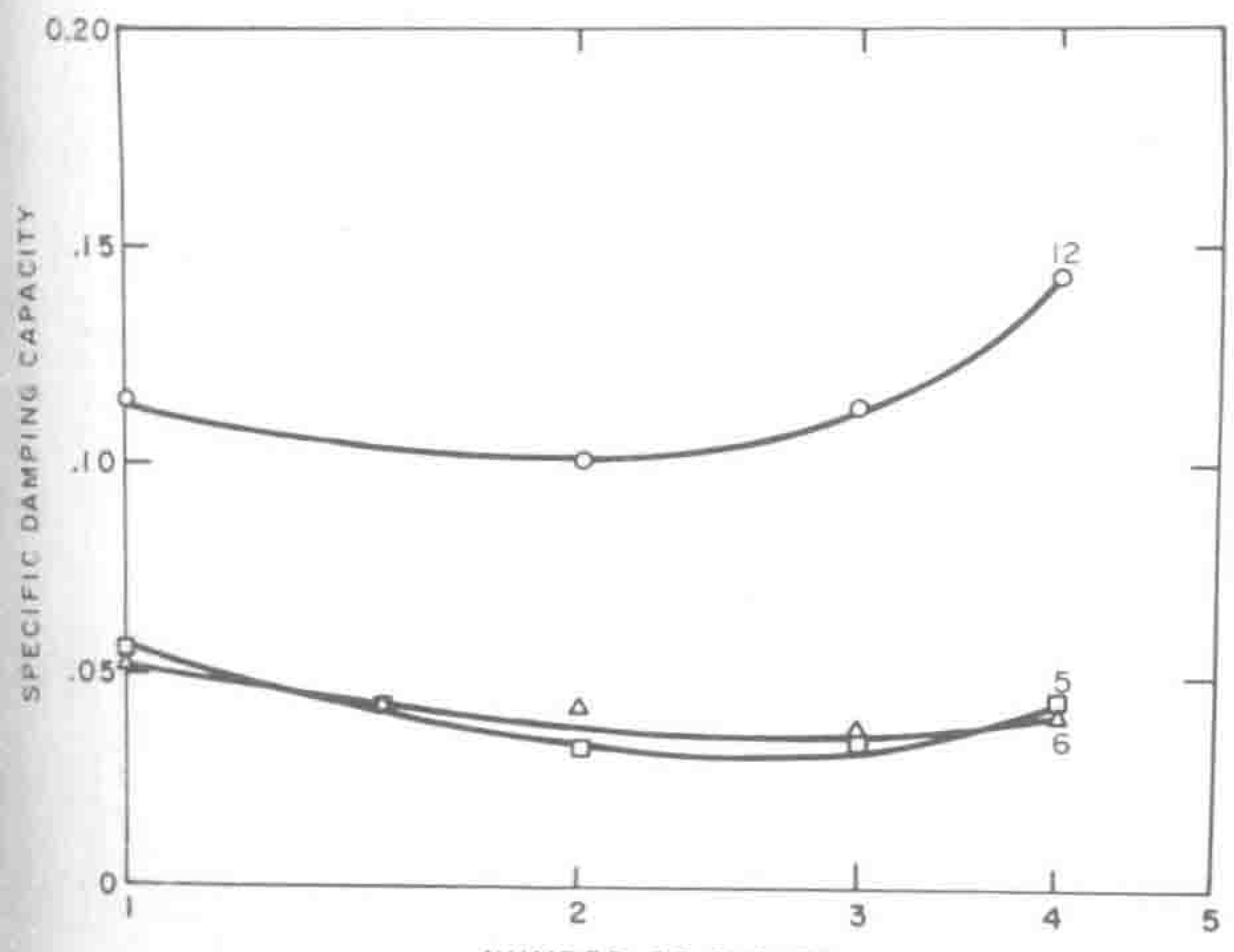


FIG. 12 — SPECIFIC DAMPING OF SALEM LIMESTONE UNDER CYCLIC COMPRESSIVE-TENSILE LOADING. THE NUMBERS IN THE CURVES REPRESENT SPECIMEN NUMBERS.

CONCLUSION

For Salem limestone, fatigue limits exist under cyclic compressive and tensile loading. Cyclic loading at a stress amplitude higher than the respective fatigue limit will positively weaken the rock specimen. The fatigue lives and limits under both cyclic compressive and tensile loading are comparable when the stresses are the same percentage of strength. However, the fatigue life under cyclic compressive-tensile loading is very much shorter than under either cyclic compressive loading or cyclic tensile loading. This behavior is highly significant because it implies that rock can be more easily fragmented through cyclic compressive-tensile loading than through either cyclic compressive loading or cyclic tensile loading.

REFERENCES

1. Burdine, N. T.: "Rock Failure Under Dynamic Loading Conditions," *Soc. Pet. Eng. J.* (March 1963) 1-8; *Trans.*, AIME, Vol. 228.
2. Cain, P. J., Peng, S. S. and Podnieks, E. R.: "Fatigue Damage in Rock by Sonic Energy," paper presented at 84th Meeting of Acoustical Soc. of America, Miami, Fla., Nov. 28-Dec. 1, 1972.
3. Grover, H. J., Dehlinger, P. and McClure, G. M.: "Investigation of Fatigue Characteristics of Rock," summary report, Drilling Research, Inc. (1950) Sec. IV.
4. Haimson, B. C. and Kim, C. M.: "Mechanical Behavior of Rock Under Cyclic Fatigue," paper presented at 13th Symposium on Rock Mechanics held at U. of Illinois, Urbana, Sept. 1, 1971.
5. Haimson, B. C.: "Mechanical Behavior of Rock Under Cyclic Loading," Annual report for ARPA contract No. H0210004, monitored by USBM, U. of Wisconsin, Madison (March 1972) 84.
6. Haimson, B. C.: "Mechanical Behavior of Rock Under Cyclic Loading," Final report for ARPA contract No. H0220041, monitored by USBM, U. of Wisconsin, Madison (April 1973).
7. Hardy, H. R., Jr. and Chugh, Y. P.: "Failure of Geologic Materials Under Low-Cycle Fatigue," paper presented at Sixth Canadian Symposium on Rock Mechanics, Ecole Polytechnique, Montreal, Canada, May 28-30, 1970.
8. Hoagland, R. G., Hahn, G. T., Rosenfield, A. R., Simon, R. and Nicholson, G. D.: "Influence of Microstructure on Fracture Propagation in Rock," Final report, ARPA contract H0210006, monitored by USBM, Battelle Memorial Institute, Columbus, Ohio (Jan. 1972) 56.
9. Peng, S. S. and Podnieks, E. R.: "Relaxation and the Behavior of Failed Rock," *Int. J. Rock Mech. and Min. Sci.* (Nov. 1972) Vol. 9, No. 7, 699-712.
10. Peng, S. S. and Johnson, A. M.: "Crack Growth and Faulting in Cylindrical Specimens of Chelmsford Granite," *Int. J. Rock Mech. and Min. Sci.* (Jan. 1972) Vol. 9, 37-86.
11. Peng, S. S. and Ortiz, C. A.: "Crack Propagation and Fracture of Rock Specimen Loaded in Compression," *Proc.*, Intl. Conf. on Dynamic Crack Propagation, Lehigh U., Bethlehem, Pa., July 10-12, 1972 (in press).
12. Lazan, B. J.: "Damping Studies in Materials Science and Materials Engineering," in *Internal Friction, Damping, and Cyclic Plasticity*, ASTM STP 378 (July 1965) 1-20.

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